Lesson 16

July 12, 2016

1. Plot the points whose spherical coordinates are $(2, \frac{-\pi}{2}, \frac{\pi}{2})$ and $(4, \frac{\pi}{2}, \frac{pi}{3})$, then convert to rectangular coordinates.
2. Change $(\sqrt{3}, -1, 2\sqrt{3})$ from rectangular coordinates to spherical coordinates.
3. Identify the surface $\rho = \sin\theta \sin\phi$.
4. Change the triple integral $\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{2-\sqrt{4-x^2-y^2}}^{2+\sqrt{4-x^2-y^2}}(x^2+y^2+z^2)^{3/2}\,dz\,dy\,dx$ to spherical coordinates.
5. Set up an integral in spherical coordinates to evaluate $\int\int\int_B (x^2 + y^2 + z^2)^2\,dV$, where B is the ball with center at the origin and radius 5.
6. Set up an integral in spherical coordinates to evaluate $\int\int\int_E 9 - x^2 - y^2\,dV$, where E is the region between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 16$.
7. Find the volume of the part of the ball $\rho \leq a$ $(a > 0)$ lying between the cones $\phi = \frac{\pi}{6}$ and $\phi = \frac{\pi}{3}$ in the first octant.
8. Set up an integral to find the volume of the solid within $x^2 + y^2 + z^2 = 4$, above the xy-plane, and below the cone $z = \sqrt{x^2 + y^2}$.

Answers:

1. $(0,-2,0)$; $(\sqrt{6},\sqrt{6},2)$

2. $(4,\frac{-\pi}{6},\frac{\pi}{6})$

3. Sphere with radius 1/2, centere dat $(0,1/2,0)$

4. $\int_0^{\frac{\pi}{2}}\int_0^{2\pi}\int_0^{4\cos\phi}\rho^5\sin\phi\,d\rho\,d\theta\,d\phi$

5. $\int_0^{\pi}\int_0^{2\pi}\int_0^{5}\rho^6\sin\phi\,d\rho\,d\theta\,d\phi$

6. $\int_0^{\pi}\int_0^{2\pi}\int_1^{4}(9-\rho^2\sin^2\phi)\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$

7. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\int_0^{\frac{\pi}{2}}\int_0^{a}\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$

8. $\int_{\frac{\pi}{4}}\frac{\pi}{2}\int_0^{2\pi}\int_0^{2}\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$